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Theoretical Aspects of the Agile Mirror

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A Planar Plasma mirror which can be oriented electronically could have the capability of providing electronic steering of a microwave beam in a radar or electronic warfare system. This system is denoted the agile mirror. A recent experiment has demonstrated such a planar plasma and the associated microwave reflection. This plasma was produced by a hollow cathode glow discharge, where the hollow cathode was a grooved metallic trench in a Lucite plate. Various theoretical aspects of this configuration of an agile mirror are examined here.

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THEORETICAL ASPECTS OF THE AGILE MIRROR

1. Introduction

Recently there has been interest in the use of a plasma sheet, whose position can be controlled electronically, as a reflector to steer microwave beams[1]. This is the so called 'agile mirror'. A recent experiment, using a hollow cathode glow discharge, has demonstrated that a sheet plasma can be formed, can reflect microwaves roughly as well as a metal plate, and can have its position electronically controlled by varying the direction of a magnetic field[2].

The experiment in Ref.(2) used a hollow cathode groove, roughly one cm deep, one cm wide and 15 cm long. This generates a high current density via the hollow cathode effect. Also in the experiment in Ref.(2) were magnetic field coils, which produced a uniform magnetic field of about 100-200 G. This field is very important in that it keeps the discharge planar. Current in the plasma flows along the magnetic field from the cathode to anode. The current in Ref.(2) was quite high, about $2\text{-}5\times10^{-1}$ A/cm^2, yet the plasma remained in the uniform glow mode and typically did not arc. However the current was limited in time to about 60 μs. More recent experiments[3] show currents typically lasting 150 μs, and in some cases, as long as the voltage pulse. This limitation is interesting although not understood at this point. Also it has not yet been verified that the plasma can be pulsed at high repetition rate, as would be needed for a fielded system.

To electronically steer this plasma along one axis, one could vary the direction of the magnetic field. This was demonstrated in the experiments of Ref.(2). To vary it along the other axis, one could envision a cathode plane with emitting elements individually addressable. By electronically selecting the set of emitting elements, one could vary the other axis of the planar reflector. Thus the configuration of Ref.(2) has great promise at this point of developing into an agile mirror for a radar or electronic warfare system.

We will see that the main role of the magnetic field is to channel the current, not necessarily to reduce ambipolar diffusion. Thus if the current can be channeled in other ways, it might be possible to operate the agile mirror without a magnetic field. One possible means of doing this is explored here.

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This memo performs a preliminary examination of several theoretical aspects of the agile mirror: the theory of the planar plasma, the electrostatic shielding of the non current carrying regions, other system considerations, and the possibility of magnetic field free operation. At this point there seem to be two possible explanations for the planar plasma. First of all, it may be a thermal plasma like the positive column of a glow discharge. Secondly it may be a beam plasma like high voltage, low density transient hollow cathode plasmas. At this point the available experimental data does not appear to provide enough information to distinguish. However the two plasmas are very different and better diagnostics could provide the necessary information. We also discuss the possible mechanisms of current production in the hollow cathode discharge. We conclude that at least as regards the simplest theoretical considerations, the agile mirror seems to be a workable concept.

2. A Uniform Thermal Plasma

The main effect of the magnetic field is to guide the discharge current so as to insure a planar configuration. The vacuum electrostatic fields of the configuration of Ref. 2 are sketched in Fig.(1). The hollow cathode is at a large negative potential. The rest of the cathode plane is an insulator, whose potential approaches ground level far from the cathode. The anode above, is a conductor at ground level. If there were no magnetic field, the electron current would follow the field lines in Fig.(1) and the discharge would not be well defined spatially. However the magnetic field guides the plasma in the vertical (z) direction. The electron orbit starting at the left most edge of the cathode hollow defines the left boundary of the current carrying region, and similarly for the right boundary. As there is no current outside this region, there is no power input and the discharge will not exist there. This boundary plays a role similar to that of a wall bounding the plasma. Electrons and ions might flow towards this boundary and out through it, but then they cool, recombine, and leave the plasma.

The actual calculation of this boundary surface is a complicated nonlinear problem, as the electron fluid element moves not only because of the electric field, but also because of the electron pressure gradients. To get a rough approximation of the divergence of the current stream line from the vertical, we assume that the configuration is two dimensional in x and z, so the electrostatic field has only these two components. For simplicity we consider the vacuum electric field configuration. Assuming that the current stream lines approximately follow the electron fluid orbits, then for the collisional, magnetized plasma, we have

$$-J_x/en \approx v_x \approx eE_x/\{m\nu_e[1+\Omega_{ce}^2/\nu_e^2]\} \qquad (1)$$

$$-J_z/en \approx v_z \approx eE_z/m\nu_e \qquad (2)$$

Here ν_e is the electron neutral collision frequency and Ω_{ce} is the electron cyclotron frequency. Thus the current stream line is given approximately by

$$dx/dz = v_x/v_z = E_x/\{E_z[1+\Omega_{ce}^2/\nu_e^2]\} \qquad (3)$$

The solution of this equation, where at $z=0$, $x=x_c$, the right hand edge of the cathode, is the current stream line which bounds the plasma on the right. Of course this is not exact because the electron velocity in the x direction is determined also by the pressure gradient, and the current stream line is not exactly the electron orbit because the electron orbit describes both electric current and ambipolar diffusion. Also, the electric field must itself be determined self consistently, and this is a complicated nonlinear problem. However we do find the expected result that as long as $\Omega_{ce}^2/v_e^2 > 1$ the plasma boundary is approximately a vertical line from the cathode to anode.

Assuming the plasma boundary is this vertical line, $x=x_c$, the normal electron and ion momentum equations give the result:

$$v_{ex}=v_{ix} =-\{(mv_e[1+\Omega_{ce}^2/v_e^2] + Mv_i\})n\}^{-1}d/dx(nT_e)$$

$$\equiv n^{-1}D_a dn/dx \qquad (4)$$

where n is the electron and ion density, and we have assumed quasi-neutrality and D_a is the ambipolar diffusion coefficient. For the parameters of Ref. 2, it is not difficult to show that diffusion is dominated by unmagnetized ion collisions, not the magnetic field. The reason is that at low ion energy ($E < 0.1$ eV), the ion charge exchange cross section in molecular nitrogen is quite large[4], about 3×10^{-14} cm^2. Thus the magnetic field forms the planar discharge by constricting the electron current, not by reducing the ambipolar diffusion. The electron velocity in the z direction is given by Eq.(2). The ion velocity in the z direction is taken as zero, since the ion current is negligible compared to the electron current except in the cathode sheath, which we will discuss later.

The electron and ion x velocity is normal to the plasma boundary. Thus the plasma is characterized by ionization, recombination and convection through the discharge boundary. In steady state, the equation for the plasma density is

$$0 = d/dx[D_a dn/dx] + An - \beta n^2 \qquad |x|<x_c \qquad (5)$$

where A is an ionization coefficient and β is the recombination coefficient. The boundary conditions are that $x=0$ is a symmetry axis and the plasma outward velocity at $x=x_c$ is the Bohm velocity, $[T_e/M]^{1/2}$. The Bohm condition is the condition for the breakdown of

quasi-neutrality; if the flow out is slower, no large sheath potential can develop to reflect the electrons back into the plasma, if it is faster, quasi-neutrality has broken down inside the plasma.

The dissociative recombination coefficient β given roughly by

$$\beta = [2.6\times10^{-8}/T_e^{1/2}][1-\exp-(3/T_e)] \text{ cm}^3/\text{s} \qquad (6)$$

where T_e is the electron temperature in eV. This formula assumes that the cross section decreases as $\varepsilon^{-1/2}$ up to about 3 eV, at which point it falls off abruptly with energy. The ionization coefficient will be discussed later.

Equation (5) must be supplemented by several additional relations before it can be regarded as a complete description of the system. First one must solve for the temperature, second, one must derive an Ohms law for the plasma and third one must derive the ionization A. (Ohms law lets us specify either the electric field or current density, but not both.) These relations involve the energy equation and the parallel part of the electron momentum equation. We will consider here a simpler system where the electron stream velocity and temperature are determined from the swarm relations for nitrogen. This data is generally obtained for plasma where geometric effects are not very important. Also it assumes that the electron density is so low that secondary processes are neglected. Hence we regard these as a qualitative rather than quantitative indication of the plasma behavior. A great deal of swarm data has been summarized by Dutton[5]. For the case of nitrogen, some typical data, summarized from Engelhardt et al[6], is shown in Fig.(2). It shows the drift velocity and electron temperature as a function of E/N, and also the characteristic momentum and energy exchange collision frequency as a function of electron energy. For the case of nitrogen, we find an approximate relation is

$$v_z(\text{cm/s}) = 8\times10^7\,[10^{14}E/N]^{5/6} \qquad (7)$$

$$T_e(\text{eV}) = 8[10^{14}E/N]^{7/16} \qquad (8)$$

where E is the electric field in V/cm and N is the gas density in cm^{-3}. The energy loss collision frequency is large compared to $(3m/M)\nu_m$ because molecular vibrational states have large excitation cross sections.

The swarm relations also give the ionization rate $A = \alpha N$. Alternatively an approximation to α based on Maxwellian electrons is

$$\alpha = 2x10^7[4\sigma/3]T_e^{1/2}\exp\text{-}[E_i/T_e] \text{ cm}^3/\text{s} \qquad (9)$$

where E_i is the ionization energy. This assumes $T_e << E_i$ and that the ionization cross section increases linearly with energy up to a maximum value of σ at $\varepsilon = 4E_i$. For N_2 σ is about equal to $2x10^{-16}$ cm^2. This formula is reasonably good for nitrogen even though the actual electron distribution function is non Maxwellian.

Generally we expect the swarm relations to be best for v_z, which depends on only momentum exchange collisions averaged over the distribution function, and quite accurate for T_e which depends on reasonably well known energy sinks. It is least accurate in deriving α, which not only depend exclusively on the tail of the distribution function, but it also neglects secondary processes.

Equation (5-9), are nonlinear and rather complicated. The treatment of the Ohm's law is simplified if one specifies the central electron density, n_o and then solves for both the electric field and current density (Ohms's law becomes an implicit relation for J and E). Then one solves Eq.(5) starting from central density n_o and $dn_o/dx = 0$ at $x=0$, and integrates to the plasma edge $x=x_c$ defined by the current boundary. The flow velocity at this boundary is given by the Bohm condition, $v=[T_e/M]^{1/2}$. This then defines a nonlinear eigenvalue problem, which in turn specifies the electron temperature T_e. Once the temperature is determined, the electric field is determined by the swarm relation, and this in turn determines v_z. The solution is completed by calculating the current $I=\langle n \rangle ev_z$, where $\langle n \rangle$ is the average electron density. In this solution, if N is constant in x, so is E, as it must be for steady state, and so is $v_z$and T_e. Calculating the properties of the thermal plasma this way, it is clear that the electron density n scales linearly as current. The scaling of n on the background density, N, cannot be obtained from only the thermal plasma. However if the plasma is governed by the conditions at the cathode, where something like an Langmuir-Child ion diode occurs, one has the classical scaling with density n is proportional to N^2.

There are two possible situations regarding the losses: recombination dominates or diffusion dominates. In the former case, the temperature is simply determined by

$$\alpha N = \beta n \qquad (10)$$

an algebraic relation for T_e. In the latter case, the eigenvalue equation becomes linear in n.

For the parameters of Ref.(2), where $n \approx 2 \times 10^{12}$, it is not difficult to show that the plasma is in the recombination dominated regime,

$$D_a/x_c^2 << \beta \qquad (11)$$

In this case, with α given by Eq.(9), we find that the electron temperature is about 2 eV. The actual value of α necessary to balance the dissociative recombination is about 4×10^{-12} cm^3/s. Since β is not strongly dependent on temperature (below about 3 eV) or non thermal tails, we regard this estimate for α as fairly accurate.

The problem is that if T_e = 2 eV, then Fig. 2 shows that the electron drift velocity is about 10^7 cm/s. For an electron density of a little less than 2×10^{12} cm^{-3}, as measured in Ref. 2, this would give a current density of about 3 A/cm^2, about a factor of 5 larger than was measured. If the temperature were 1 eV, the swarm relations would give about the right current density.

We examine now whether any secondary ionization process can substantially increase α at low temperature. Brunet et al[7,8] have examined possible enhancements to α from a large number of electron secondary processes including superelastic collisions from vibrational excitations, and electron impact ionization of metastables. All of these processes can greatly increase α when it has very low values. However once α gets above about 10^{-13} cm^3/s, as it must in the discharge in Ref 2, none of these processes have a significant effect.

The process to which Brunet et al attribute enhanced ionization, at high ionization rate, is associative ionization of metastables. To understand this process we consider a very simple model, where the

single metastable state considered is the N_2 metastable state denoted $a'\Sigma_u^-$, which has an excitation energy of 8.4 eV. A collision of two of these metastable states has sufficient energy to ionize the molecule in the reaction

$$N_2(M) + N_2(M) \rightarrow N_2 + N_2^+ + e$$

The cross section of this process is very large, around 10^{-14} cm^2 according to Refs. 7 and 8. The rate equation for the metastables is then given by

$$dN(M)/dt = \alpha_x nN - \alpha_{dx} nN(M) - \alpha_M N(M)^2 - \alpha_d N(M)^2$$

$$- \alpha_{dN} N(M)N - N(M)/\tau_M \qquad (12)$$

Here α_x is the excitation rate for metastables due to electron atom collisions, α_{dx} is the de-excitation rate due to collisions of metastables with electrons, α_M and α_d are the loss rates of metastables due to ionization and de-excitation of metastables by metastables, α_{dN} is the de-excitation rate due to collisions with ground state molecules, and τ_M is the spontaneous decay rate of the metastable. For convenience we have dropped the subscript in N_2.

The last two terms in Eq.(12) are very small[9], and we neglect them. We assume that α_x is given by an expression like Eq.(9), but with E_i replaced with the metastable energy E_M. We further assume that the maximum cross section for metastable generation is about 10^{-16} cm^2 as it is for ionization. Assuming that α_{dx} is about the same, electron de-excitation can be neglected since $N(M) << N$. For α_M, Brunet quotes a rate of 2×10^{-10} cm^3/s. Assuming that α_d is about the same, we find that the steady state number density of metastables is given roughly by

$$N(M)^2 \approx \alpha_x nN/2\alpha_M \qquad (13a)$$

and the corresponding electron production rate in the electron equation is then equal to

$$dn/dt = ... + \alpha_x nN/2 \qquad (13b)$$

Note that the production rate has exactly the same form as originally, except that the coefficient is half the metastable production rate rather than the direct ionization rate. Since the metastable energy is about half the ionization energy, the plasma can be maintained at about half of the electron temperature, or at about one electron volt instead of two. For the parameters of Ref. 2, $n=2x10^{12}$ and $N= 4x10^{15}$ cm^{-3}, so the steady state metastable density is given roughly by $N(M) \approx 10^{13} cm^{-3}$. Since the metastable energy is 8.4 eV, the metastable energy corresponds to a net increase in average energy per particle of only about 0.02 eV.

Thus the secondary production of the $a'\Sigma_u^-$ metastable state in the N_2 gas is a very likely explanation for its greatly enhanced ionization. For these metastables to be produced quickly enough, there must be a transient production mechanism. If the electron temperature were at the equilibrium value, the production time for the metastables would be much too long to explain the results of Ref. 2. When the voltage there is turned on, the field in the plasma is very high, initially about 100 V/cm. Thus initially the electron temperature, and therefore the ionization rate and metastable production rate are all much higher than their equilibrium values. Hence the ionization and metastables can be produced very quickly. However, at equilibrium, the electric field in the bulk is about 1 V/cm, and virtually all of the voltage drop is across the cathode sheath. Thus, in the experiment of Ref. 2, it appears that steady ionization is governed by metastable collisions, and that initial transients can set up the necessary electron and metastable populations quickly.

3. A Beam Produced Plasma

At kilovolt energy, the momentum exchange mean free path is much longer than the 15 cm length of the plasma, and the energy loss mean free path is slightly longer. Thus, if an electron beam, which carries the current is somehow produced in the cathode region, it could propagate across the plasma and ionize the background. We assume that all of the voltage drop is near the cathode, and it somehow generates a beam. Let us start by calculating the energy loss of the beam and the ionization balance.

To calculate the energy loss of the energetic beam, we use Bethe's formula,

$$dE/dz = [2\pi NZe^4/E]\ln\{4E/\langle E_x\rangle\} \tag{14}$$

where E is the electron energy, Z is the number of electrons per molecule, we assume Z=30 for air, and $\langle E_x\rangle$ is the average energy various excitations and ionization of the molecule, we assume $\langle E_x\rangle=10$ eV. Neglecting the dependence of the logarithm on E, we can integrate Eq.(14) across the plasma of length L to get that the total energy loss of the beam is given by

$$\Delta E(\mathrm{eV}) = 1.5\times10^{-14}N(\mathrm{cm}^{-3})L(\mathrm{cm})/E_o(\mathrm{eV}) \tag{15}$$

where we have taken a value of 6 for the logarithm, corresponding to the 1.5 keV electron energy used in Ref. 2. Also we have assumed $\Delta E \ll E_o$, E_o being the input energy, 1.5 kV in Ref. 2. To calculate the ionization, we must count not only the direct ionization, but also the ionization generated by the secondary electrons, since many of them are generated with energy above the ionization energy of air. These values have been tabulated[10]. For nitrogen, the energy to generate each electron ion pair is 34 eV. Thus the total energy loss for the electron, divided by 34 eV gives the total number of electron ion pairs produced by each beam electron as it transits the plasma. Since this is proportional to the length L, we can translate this into volume production rate, which is 3×10^3NJ where J is the current density in A/cm^2 and we have assumed E_o=1.5 keV.

The electrons are lost by dissociative recombination, discussed in the last section. As we will discuss, the plasma temperature is somewhat under an electron volt, so we take for the dissociative

recombination rate $5x10^{-8}n^2$. Balancing the beam production with dissociative recombination, we get the result

$$3x10^3JN = 5x10^{-8}n^2 \quad (16)$$

where we have assumed the beam energy of 1.5 keV. Thus for the parameters of Ref.2, the electron density is about 10^{13}, easily enough to reflect the microwave radiation.

Now let us discuss the plasma produced. The primary and secondary electrons produced by the beam ionization will have some spectrum of energy form about half of the beam energy on down. However these electrons cool very rapidly by exciting various electronic and vibrational states of the air molecule. Shown in Fig.(3), taken from Ref.(4) is the momentum exchange and energy exchange electron collision cross section for N_2, O_2, and O as a function of energy for energies below 100 eV (where Bethe's formula is no longer valid). Above an electron volt, the greatest slowing down time is at an energy of about 7 eV, where it is several hundred nanoseconds, still comparable to or greater than the recombination time. However, even here the electron does not have to slow down very much before it begins to excite the vibrational states and then slows very rapidly again. Thus, while there may be something of a pileup of electrons at 5-7 eV, most of the electrons are at or below 1 eV, where their cooling rate is very low. At this point they are lost by dissociative recombination. Hence the bulk temperature for a beam ionized plasma is quite low, below 1 eV.

The question now is how to tell whether the plasma is a thermal plasma or a beam plasma (or combination). While there does not appear to be sufficient information to tell at this point, these plasma are very different in their properties and scaling. For the beam plasma, the electron density scales as $J^{1/2}$ and $N^{1/2}$, whereas for the thermal plasma it scales as J and N^2. For the beam plasma, if the anode were a screen, or had a small hole in it, one could observe energetic electrons emerging, whereas of course this would not be possible for the thermal plasma. For the beam plasma, the voltage would increase with length, since the energetic electrons need the extra energy to get across the longer plasma. For the thermal plasma the voltage drop is nearly independent of length; that is a fixed large voltage drop occurs near the cathode sheath, and a very small drop occurs in the bulk plasma, we have seen that just a few volts per centimeter are sufficient to maintain the thermal plasma.

Correspondingly, the beam plasma will have no electric field in the main plasma; we have seen that a voltage drop across the main plasma of no more than a few percent of the overall voltage drop will produce and sustain the thermal plasma, especially if there is initial ionization. Thus the beam plasma will have virtually no electric field while the thermal plasma would have an electric field of a few V/cm. Another important difference is that the thermal plasma requires metastables for maintenance while the beam plasma does not. It may be possible to try the experiment in a gas with no metastables. Some of the properties of the beam and thermal plasma are summarized in Table 1. The last line of the table will be discussed in Sec 7.

4. Shielding in the Current Free Regions

The agile mirror is defined in terms of a current path specified by the magnetic field and the emitting portions of the cathode. To terminate the discharge, we rely on dissociative recombination in the diatomic gas. Since the rate of this process is proportional to n, and becomes small for low electron density, seed electrons from previous realizations of the agile mirror will be spread throughout the gas volume. The question is why these electrons do not avalanche in the imposed electric field and cause breakdown throughout the chamber. The key is that while seed electrons and electric fields are initially present, there is no current carrying capability except in the defined region. This limits the electron production in these other regions.

To estimate the maximum breakdown in the non-current carrying regions, we assume a very simple one dimensional configuration. Assume that the gas is between two non conducting parallel plates which at t=0 have a voltage drop V across them. Initially this voltage is a kilovolt or two and the electric field is perhaps 50-100 V/cm, the values used in the experiment. This electric field causes electron production, but it also causes electron drift. Since the plates are nonconducting, the drifting electrons pile up at the anode plate, and the ions pile up at the cathode plate, and form a space charge region. This electron space charge works to cancel the imposed electric field and thereby reduce electron production.

We will give here a simple one dimensional theory of the space charge cancellation and a rough estimate of the upper limit to the electron density. To simplify, we neglect recombination, so the electron density and drift velocity are governed by

$$dn/dt = \alpha n N \qquad (17a)$$

$$v = -eE/mv_m \qquad (17b)$$

In one dimension, the change in electric field is determined by the displacement current,

$$dE/dt = 4\pi nev = -4\pi ne^2E/mv_m \qquad (18)$$

A simple estimate for an upper limit to the electron density can be obtained by calculating α from the temperature equation,

$$dT/dt = (2/3)[mv^2\nu_m - \nu_u T_e - E_i\alpha N] = 0 \qquad (19)$$

where E_i is the ionization energy and ν_u can be obtained from Fig.(2) and Ref.(4). An upper bound for the temperature and ionization rate can be determined by balancing the collisional heating with energy loss due to ionization alone. This gives a solution for α

$$\alpha = e^2E^2/m\nu_m E_i \qquad (20)$$

Using this expression for α in the density equation, we find that the system is governed by two equations for E and n

$$dn/dt = e^2E^2n/m\nu_m E_i \qquad (21a)$$

$$dE/dt = -4\pi ne^2E/m\nu_m \qquad (21b)$$

Assuming that the initial density and final electric field are both zero, we find that the final density n_f can be calculated in terms of the initial electric field E_o as

$$n_f = E_o^2/8\pi E_i \qquad (22)$$

This shows that the maximum electron density is obtained by converting the initial electric field energy to ionization energy. Since this is an upper limit for the final electron density, it would actually be less than this value, probably considerably less since generally $\nu_u T_e \gg E_i\alpha N$. If we assume that E_o is 50 V/cm and E_i is 15 eV, we find that an upper limit for the electron density in the regions where no steady state current flows is about 5×10^7 cm^{-3}. This is sufficiently small that it should have virtually no effect on the propagation of microwaves.

In the more realistic case of a two-dimensional configuration, electrons and ions flow along the magnetic field lines until they charge the cathode plate so that the potential drop along the field is zero or less than the potential needed to generate breakdown. The actual self consistent, two-dimensional calculation appears to be very complicated.

5. Plasma Limitations on System Performance

The plasma reflector potentially limits the performance of a radar system in at least three ways. First of all, the transmitted radar beam must be sufficiently weak that it does not itself produce significant ionization in the plasma. Secondly, the microwave noise emitted by the plasma reflector must be sufficiently small that it does not overwhelm the much weaker received signal. (For the case of a transmitter alone, perhaps in an electronic warfare system, this second consideration is not a constraint.) Thirdly, the plasma may heat the neutral gas and change its properties during the pulse. Fortunately, none of these place a severe limit on the radar system performance.

Let us first consider the case of ionization by the microwave pulse itself. Because the frequency of the microwave radiation is so high, this turns out to be a small effect. To calculate this, we compare the heating from the microwave field with the heating from the dc field which produces the discharge. The heating per unit volume from the dc field is given by $ne^2E_o^2/mv_m$. Since the cooling rate for the electrons depends linearly on N, the dc breakdown conditions specifies an E/N. Since $v_m << \omega$ (the microwave angular frequency), the heating by the microwave field is given by $ne^2v_mE_\mu^2/2m\omega^2$, where E_μ is the amplitude of the microwave field. Equating the microwave heating rate at breakdown with the dc heating rate there, we find that the microwave field strength for breakdown is independent of N and is given roughly by

$$I_b(\mathrm{W/cm^2}) < 125(f/10\mathrm{GHz})^2 \qquad (23)$$

where f is the frequency, $\omega/2\pi$ and we have assumed that the constructive interference between the transmitted and reflected radiation increases the field amplitude at the interference maxima by a factor of two, and increases the maximum irradiance by a factor of four. For 10 GHz microwave radiation, $\omega=6\times10^{10}$, so that the irradiance can be over 100 W/cm^2 before plasma production by the radar beam itself becomes a consideration. Fortunately, most radar systems do not have such a high irradiance at the transmitting antenna.

Let us now consider the thermal emission of the plasma. This is important for the received pulse only. The potential problem is

that a 1 eV plasma reflector radiates as a black body, it will be by far the largest noise temperature in any radar receiver system. Fortunately, however, a plasma radiates as a black body at the frequency in question only if it is optically thick. That is if the absorption length of the radiation is much smaller than the characteristic size of the plasma.

The region of the plasma mirror below the critical density must be optically thin to operate, or else there would be significant absorption of the radar wave. The absorption of the plasma is determined either by the round trip damping of the radiation as it propagates from vacuum to the critical surface and back to vacuum; or else by the absorption in the overdense plasma, by the evanescent wave, if the thickness of the underdense plasma is very short. These absorption's have been tabulated Ref.(1).

$$F = \max\{8L\nu_m/c,\ 2\nu_m/[\omega_p^2 - \omega^2]^{1/2}\} \tag{24}$$

where L is the distance between the critical surface and vacuum assuming a linear density profile. Kirchoff's law then gives the radiation temperature as[11]

$$T_r = FT_e \tag{25}$$

Thus as long as F is very small, the thermal antenna temperature of the plasma reflector is much less than the electron temperature. For reasonable values of L, ν_m, and ω, the radiation temperature of the mirror can be comparable to or less than room temperature. For instance for $L=1/2$ cm and $\nu_m = 3\times10^8 s^{-1}$, the radiation temperature of the antenna is about 250°K. Thus as long as the plasma reflector is at thermal equilibrium, its radiation temperature is not necessarily higher than any other component of the receiver system.

Finally let us consider the heating of the bulk gas in one plasma pulse. The total power input into the plasma is IV_c where I is the current and V_c is the voltage across the uniform glow region. We have seen that for a 100 mTorr gas, the field is somewhat under 1 V/cm, so the total voltage drop across the uniform glow is only about 15 V. For I= 4 A (the average over the positive half of a sine wave with 7 A maximum current) for 50 μs, the total input is about 2×10^{-4}J. The total number of particles is 2.5×10^{15} particles per cm^3 times the volume, 225 cm^3, or about 6×10^{17} particles. Thus the total

input energy is about 3×10^{-22} J per particle. This corresponding to a total temperature rise of 10^{o} C if all input energy directly heats the particles, assuming no losses and three directional and two rotational degrees of freedom for the linear diatomic molecule. Because there is also thermal conduction and radiation, the temperature rise will be somewhat less than this. Thus, heating of the bulk gas seems to be relatively unimportant, specifically, there does not seem to be any way it could be responsible for the current termination observed in Ref. 2. However dissipation processes around the hollow cathode, where the fields are much higher could be playing an important role. We will discuss this qualitatively in Section 7.

6. A Possible Magnetic Field Free Configuration for the Agile Mirror

We have seen that the effect of the magnetic field on the discharge is to constrain the electron current so that it flows along a predetermined axis even though the electric field lines are diverging. A possible alternate approach is to keep the electric field lines from diverging, so they are guided and straight. If this could be done, the magnetic field might not be required.

One possible way to do this is to have the anode segmented by insulators as shown in Fig. (4a), so that there are small conducting regions electrically isolated from one another. For the cathode, one does almost the same thing, except that on each conductor, there is also a small volume, dense plasma source, perhaps a spark discharge as shown in Fig (4b). This produces an initial dense plasma, having number density n_d on the conducting portion of the cathode. In doing this, the potential between cathode and anode would be less than that required for breakdown with cold electrodes. Hence by selecting the plasma sources which fire, one selects the intersection of the plasma reflector with the cathode surface. Behind each segment is a switch, which would be open circuited unless the cathode plasma were generated there. Thus only those selected portions of the cathode can conduct current.

One now not only generates a voltage between the cathode and anode, but also controls the voltage from one conducting section to the next on both the cathode and anode plane. In this way, the shape and direction of the electric field lines and electrodes may be controlled. Since a cathode plasma is preformed in this proposed scheme, there is not necessarily a cathode sheath any more. Instead it is the dense plasma which supplies the electron current. As current is drawn from this dense plasma, it pulls away from the cathode, and eventually the process terminates. We now consider an approximate theory of this process.

We will consider a one-dimensional configuration where there are two planar electrodes a distance d apart. On the cathode at $t=0$ is a dense plasma of density n_d and thickness $r \ll d$. At $t=0$, a voltage V_0 is turned on between the plates, and a main plasma of density $n(t)$ begins to form within the gas of density N. As we have seen, the voltage to maintain this plasma is quite low if there are no considerations of voltage drop across a cathode sheath.

The electron current must be continuous as one goes from the dense plasma to the main plasma, so

$$n_d v_d = nv \qquad (26)$$

where v is the electron velocity in the main plasma and v_d the electron velocity in the dense plasma. Thus the electron velocity in the dense plasma is very much less than that in the main plasma. The v in the main plasma (as well as the electron temperature) is assumed given by for instance the swarm data. The cathode is assumed not to be an emitter, but it can accept incident ion current. The electrons then pull away from the cathode leaving an ion sheath. The width of this evacuated region is denoted s. This s is assumed to be so small that the ions are collisionless in traversing it. As the electrons evacuate this region, the ions are pulled into the cathode, so that the ion current $J = n_d e v_d$, where $v_d = ds/dt$. This ion current must be limited by the space charge limiting current, so

$$J(A/cm^2) = 10^{-8}(M_a/M)^{1/2} V(\text{volts})^{3/2}/s(\text{cm})^2 \qquad (27)$$

where V is the voltage across the gap. Thus as the ions pull away, a voltage forms across the gap of width s. This and the voltage across the plates are related by

$$V_o = Ed + V \qquad (28)$$

Here E is the electric field in the main plasma. Thus the equations describing the system are Eq. (27, 28) and

$$ds/dt = (n/n_d)v_z \qquad (29)$$

$$dn/dt = \alpha nN - \beta n^2 \qquad (30)$$

$$J = n_d e[ds/dt] \qquad (31)$$

$$V = [10^8 Js^2(M/M_a)]^{2/3} \qquad (32)$$

Equations (28-32) are completed by the expressions for v_z, α and β in terms of E/N.

These relations can be solved numerically, but the qualitative behavior is not difficult to see. As the voltage is turned on, the main gas begins to ionize and draw a current. However this current pulls

the dense plasma away from the cathode a distance s. To draw the current to the cathode across this gap, a voltage, specified by the ion diode relation, is set up. This reduces Ed until the plasma can no longer be maintained. Let us estimate the n_d required for 100 μs lifetime. If the maximum voltage is 1 kV and the current is 0.3 A/cm^2, then the dense plasma pulls away from the cathode 300μm according to Eq.(27). If the main plasma has a density of $3x10^{12}$, the electron carrier velocity is about 10^6 cm/s. Then for the dense plasma to go 300μm in 100 μs requires that its density be about 10^{16} cm^{-3}.

Hence with a very different scheme for plasma production and specification, it may be possible to produce agile planar plasma reflectors without using a magnetic field to guide the current. Relatively simple experiments could be tried to test the validity of this proposed scheme.

7. Plasma Current and the Hollow Cathode Region.

The most complicated part of the discharge produced in Ref.(2) is undoubtedly the hollow cathode region. The hollow cathode allows much greater current density than a conventional, cold cathode. This is the "hollow cathode effect". The hollow cathode effect seems to be different depending on whether the plasma is steady state[1,2,3] or transient[12]. The theory of the hollow cathode plasma is very difficult because it is non fluid, but collisional, and inherently two or three dimensions. Here we confine ourselves to very qualitative analyses.

As most of the voltage drop is across the hollow cathode region, a natural question is whether energetic electrons are produced. The electrons in the hollow cathode, at energies of a few hundred ev to perhaps a kilovolt are relatively collisionless. Figure 5 shows the momentum exchange cross section[14] for N_2. At 500 eV, the mean free path is about 10 cm in 100 mTorr nitrogen, and is perhaps longer if the background density is reduced by air heating in this region. Thus reflexing electrons are present and could be playing an important role. In the cathode hollow, there is most likely a hot electron plasma, consisting in part of these reflexing electrons. These reflex perhaps 10 or more times before slowing down and joining the thermal plasma.

The ions however are collisional. The charge-exchange cross section for an ionized oxygen or nitrogen molecule with its parent neutral is about 2.5×10^{-15} cm^2, again reasonably independent of energy from a few ev to about a kilovolt[15]. Even if there is considerable gas heating, the ions are still collisional.

In the absence of strong cathode electron emission, the current is carried by ions, and for a planar cathode, the maximum ion current density is given by the space charge limited current assuming the ions are collisional with constant mean free path λ. For gap spacing s and voltage V, this current density is given by

$$J(\mathrm{A/cm^2}) = 2.3(\lambda/s)^{1/2} J_{LC} \qquad (33)$$

valid for $2.3(\lambda/s)^{1/2} < 1$. Here J_{LC} is the space charge limited current for a collisionless Langmuir Child ion diode given in Eq.(27)

To see the hollow cathode effect in steady state, let us look at some data by Little and von Engel. They looked at a steady state air plasma with two planar cathodes that face one another and are parallel to the axis of the main discharge. The cathodes can be moved closer together or further apart. Also the pressure in the gas can be varied. Cathodes very far apart act as two separate cathodes and there is no hollow cathode effect. However as they are moved closer together, the individual cathode falls interact, and the current greatly increases. A plot of current versus ap, the cathode separation times the gas pressure, from Ref.(12) is shown in Fig.(6). The hollow cathode effect is apparent at small ap.

The structure of the electric field near the cathode has also been measured in Ref.(12). A plot of electric field (proportional to the displacement Δ of a probe beam) versus distance from the cathode plane is shown in Fig.(7) for the case of the pressure of 300 mTorr. For the solid curve, the voltage is 376 V and the current density is 6×10^{-3} A/cm^2. For the dashed curve it is 318 V and 2×10^{-3} A/cm^2.

To see the hollow cathode effect, let us compare the measured current density with that predicted by the collisional ion diode relation, Eq.(33). Taking the measured values for voltage and current, the collisional ion diode would predict a current density of 3×10^{-4} A/cm^2. Thus the measured current density is 20 times larger.

It seems as though there has to be more than just ion current in order to explain the results of Little and von Engel. There must be additional electron current. Also, since the experiment is steady state, the current must come from electrons emitted by the cathode. There is no other possibility since the steady-state feature rules out displacement current or other transient processes such as those discussed in Sec. 6. The only source of steady-state electron current in a hollow cathode discharge would appear to originate in electron emission from the cathode due to bombardment by ultraviolet photons. However the photons have to be fairly energetic in order to emit electrons with reasonable quantum efficiency. In Fig.(8), from Chapman[16], is shown a plot of quantum efficiency as a function of photon energy and material. For photons with energy of 10 ev and higher, quantum efficiencies are about 10^{-1} for a large number of materials.

Excitation cross sections for N_2 and O_2 are listed in Banks and Kockarts[17]. We estimate from Ref. 17, and Fig. 3, that the average cross section, summed over all transitions with energy greater than 10 ev, is about $2x10^{-16}$ cm^2. Approximating the rate as the cross section times the thermal velocity of the electron at say 300 eV, we find an approximate expression for the production of photons with E>10ev, per unit volume as about

$$R(E_{ph}>10eV)/cm^3s \approx 2x10^{-7}nN \tag{34}$$

In the experiment of Ref.(2), the measured current density is typically about 0.2-0.5 A/cm^2, far in excess of that given by Eq.27 with V=1 kV and a gap of half a centimeter. Let us see whether the uv photons might be responsible for the enhanced cathode emission in Ref.(2). Assume the reflexing electrons have energy 300 ev and reflex 30 times before joining the main plasma. Then to provide 0.5 A/cm^2, these electrons would have a density of about 10^{11} cm^{-3}. Thus if $N=4x10^{15}$, about $2x10^{19}$ photons above 10 eV are produced per cubic centimeter per second. Assuming a quantum efficiency of 0.1, the current density this radiation can generate is about 800 mA/cm^2, since all dimensions of the hollow are about a centimeter in Ref. 2. This calculation, while very qualitative is reasonably consistent with the data.

There are at least two possible simple experimental tests of this proposed mechanism. First, one can simply look for these photons by looking along the cathode hollow. Secondly, one could try the experiment with a cathode material such as aluminum which, according to Fig. 8 has a very low quantum efficiency and could not effectively generate photons.

Let us consider the effect of the hollow cathode on whether a thermal plasma or beam is generated. In either case, there must be an emission mechanism from the cathode, and we assume this is uv bombardment. Thus, in either case, there must be emission from the side walls to form the cloud of reflecting electrons, which in turn generate the uv. However electrons emitted from the center of the bottom wall would not reflex, but would accelerate straight down the axis to form the beam. Thus if the plasma is a beam plasma, it seems likely that most of the current is emitted from the bottom wall; if it is a thermal plasma, most of the current would be emitted from the sidewalls. One could test this hypothesis by running a discharge in a

hollow cathode configuration with no bottom wall. This is listed in the last line of Table 1.

Another possible explanation for the current in Ref. 2 is that the generation mechanism is inherently transient. The basic mechanism was described in Section 6. Instead of generating a dense plasma, it might be that in the configuration of Ref. 2, the dense plasma is spontaneously generated by the breakdown mechanism. The required density is at least 10^{16} cm^{-3}, larger than the ambient gas density, so it would have to be generated by material spalled off the cathode, most likely around the sharp edges where the fields are enhanced. If this plasma is generated by the breakdown mechanism, it could be measured by laser interferometry. For a 10^{16} cm^{-3} plasma, and a 15 cm path length, there would be a large phase shift for either CO_2 or Nd laser light propagation. Alternatively this effect could be investigated by generating the dense plasma in some other controlled manner, not necessarily dependent on cathode geometry, as discussed in Sec.6.

An important issue in the future development of the agile mirror is just how to provide the electron current from the cathode if a different mechanism than the hollow cathode turns out to be required. Let us consider a number of possibilities. For electronic steering we require a cathode plane with conducting elements individually addressable. One possibility is to use a large number of individually addressable hollow cathodes, each one about a centimeter or so in diameter. This configuration would be the most similar to that used in Ref.(2), but it is one where the emitting region on the cathode plane could be selected electronically. Another possibility would be use of a large number of small plasma guns along the cathode plane, as discussed in Sec. 5. Analogously, one could use a number of sharp points on the cathode surface, again individually addressable, and let the plasma generate from the geometrically enhanced field there. The Russians have used such a scheme for generating high quality electron beams of 100μs duration[18].

Another possibility is the use of vacuum field emitters, which are like the sharp points, but on a microscopic scale. Work in this area has been pioneered by SRI[19]. These field emitters can emit the current required for the agile mirror, 0.1 A/cm^2 over a large area. However one difficulty with them is that they work only in very high vacuum, of order 10^{-9} torr. When the pressure is increased to 10^{-5}

torr, they still work, but with degraded performance. Even in this 'high pressure regime', as SRI calls it, the pressure is nearly 5 orders of magnitude lower than that used in Ref.(2). If these field emitters can work at higher pressure, or else if very different pressures in the plasma region can somehow be maintained, the vacuum field emitters could be a very intriguing possibility for the current source. Finally there is the possibility of a hot cathode. These routinely emit for long periods of time at 10 A/cm^2, at least an order of magnitude greater current density than that required for the agile mirror. However they typically require high vacuum (10^{-5} torr or better) for long life operation. Using a thermionic cathode in a 10^{-1} torr gas would undoubtedly greatly constrain the choice of both emitter and gas. However if such a choice does exist, it could be a very simple solution to the problem of electron emitter.

8. Discussion:

We have analyzed a number of features of the agile mirror plasma. The role of the magnetic field is to channel the current, not to retard ambipolar diffusion or confine the plasma in any way. It seems almost certain that secondary ionization processes, probably metastable-metastable collisions, are very important for the maintenance of the main sheet discharge if it is a thermal plasma. It is also possible that the hollow cathode region generated a beam at nearly the full voltage and the sheet plasma is a beam generated plasma. At this point the experiments cannot rule out either possibility. By properly specifying the electric field magnitude and direction, and using preformed trigger plasmas on the cathode surface, it may be that the magnetic field could be eliminated, making the system simpler and lighter. The most vexing theoretical problem is the structure of the hollow cathode plasma, and analogously, one of the most important technical aspects of the agile mirror is how the current is provided at the cathode. Generally, these initial theoretical analyses indicate that the agile mirror is a workable concept.

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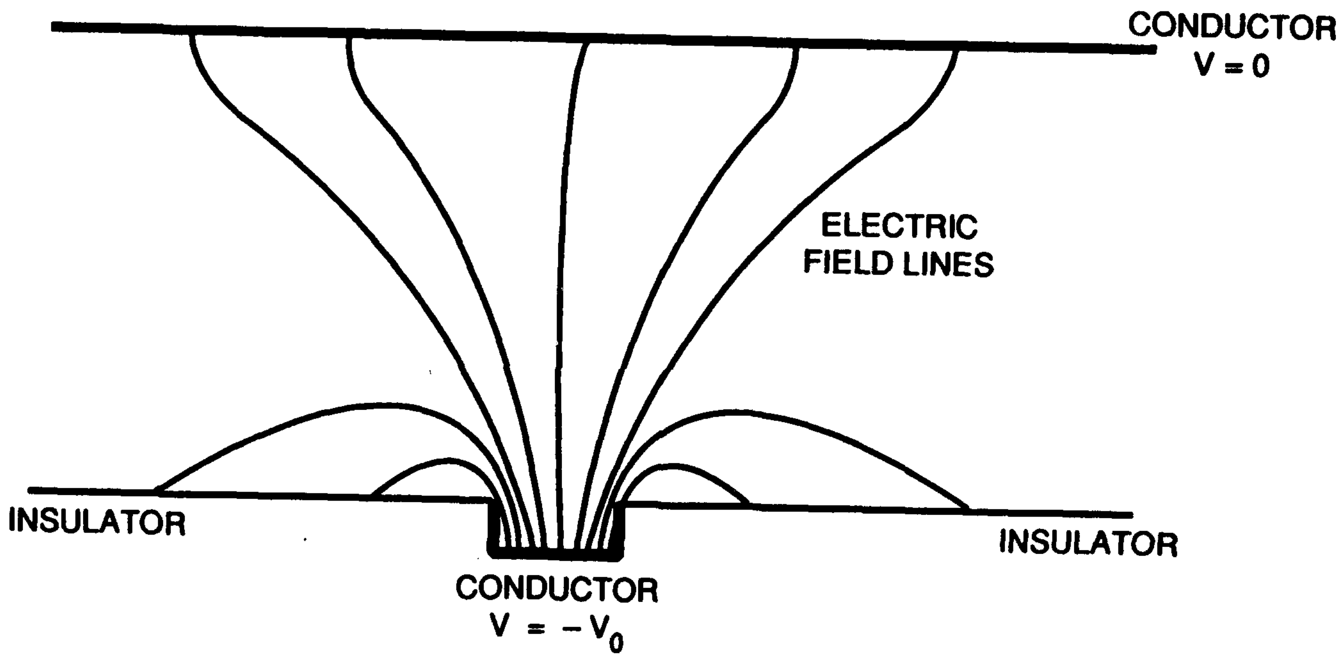

Fig. 1. The vacuum electric field configuration for the hollow cathode discharge configuration of Reference 2.

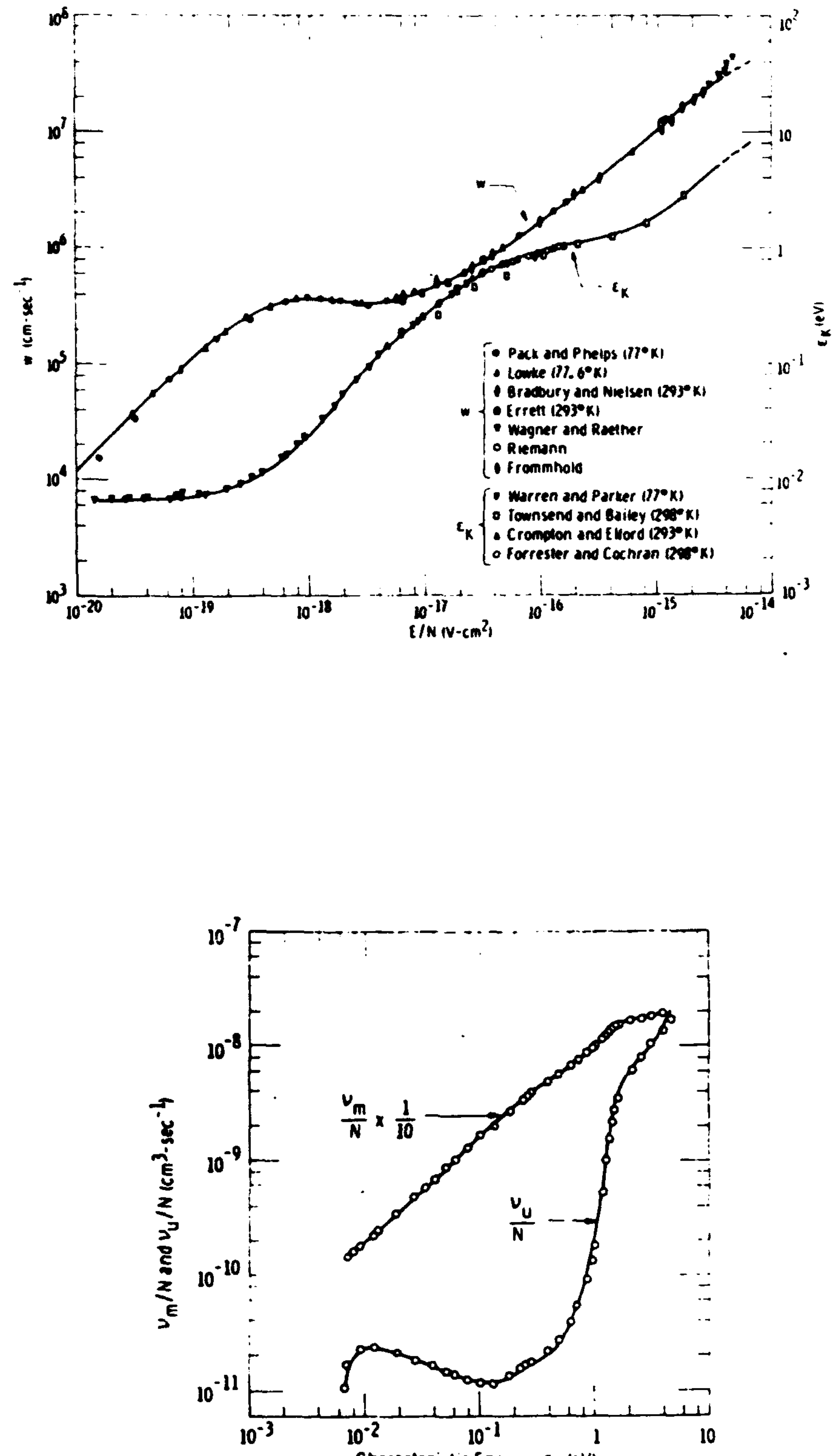

Fig. 2. Data for nitrogen from Ref 4, top: Drift velocity and characteristic energy (Electron temperature) as a function of E/N; bottom: Momentum and energy exchange collision frequency as a function of energy, b).

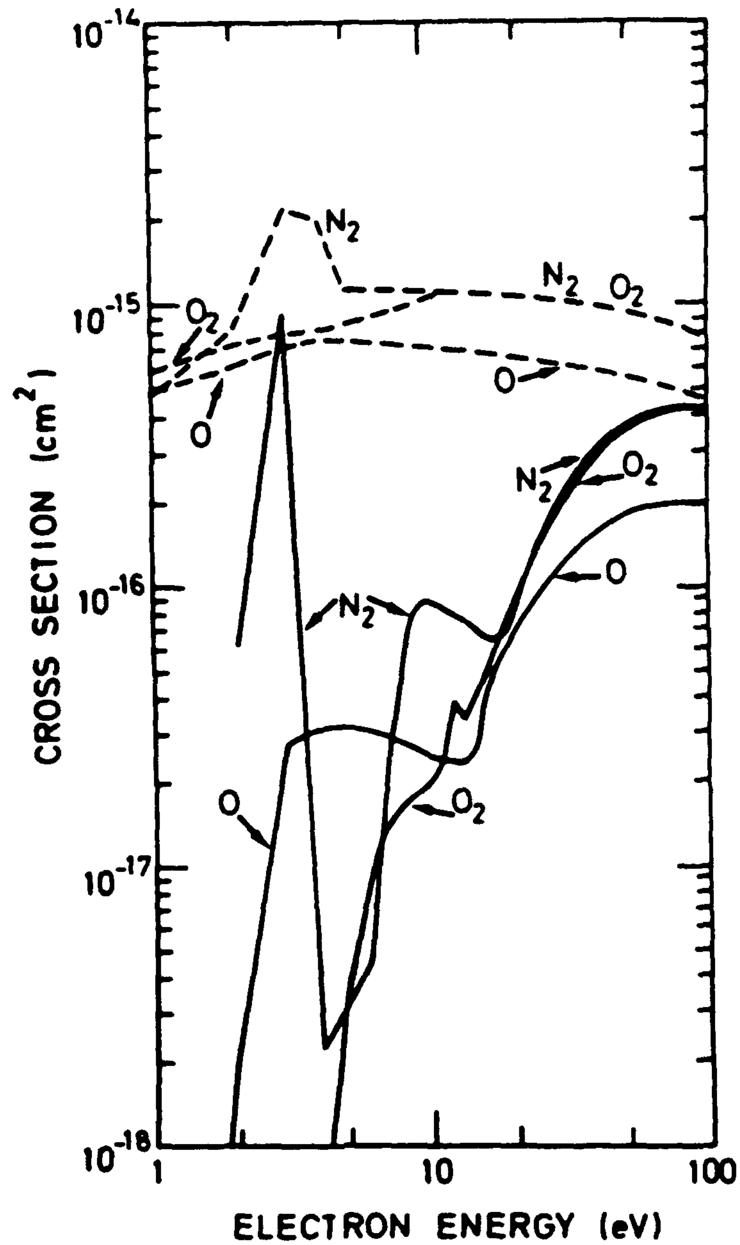

Fig. 3. Momentum and energy loss cross sections for N_2, 0_2 and 0 from Ref. (4).

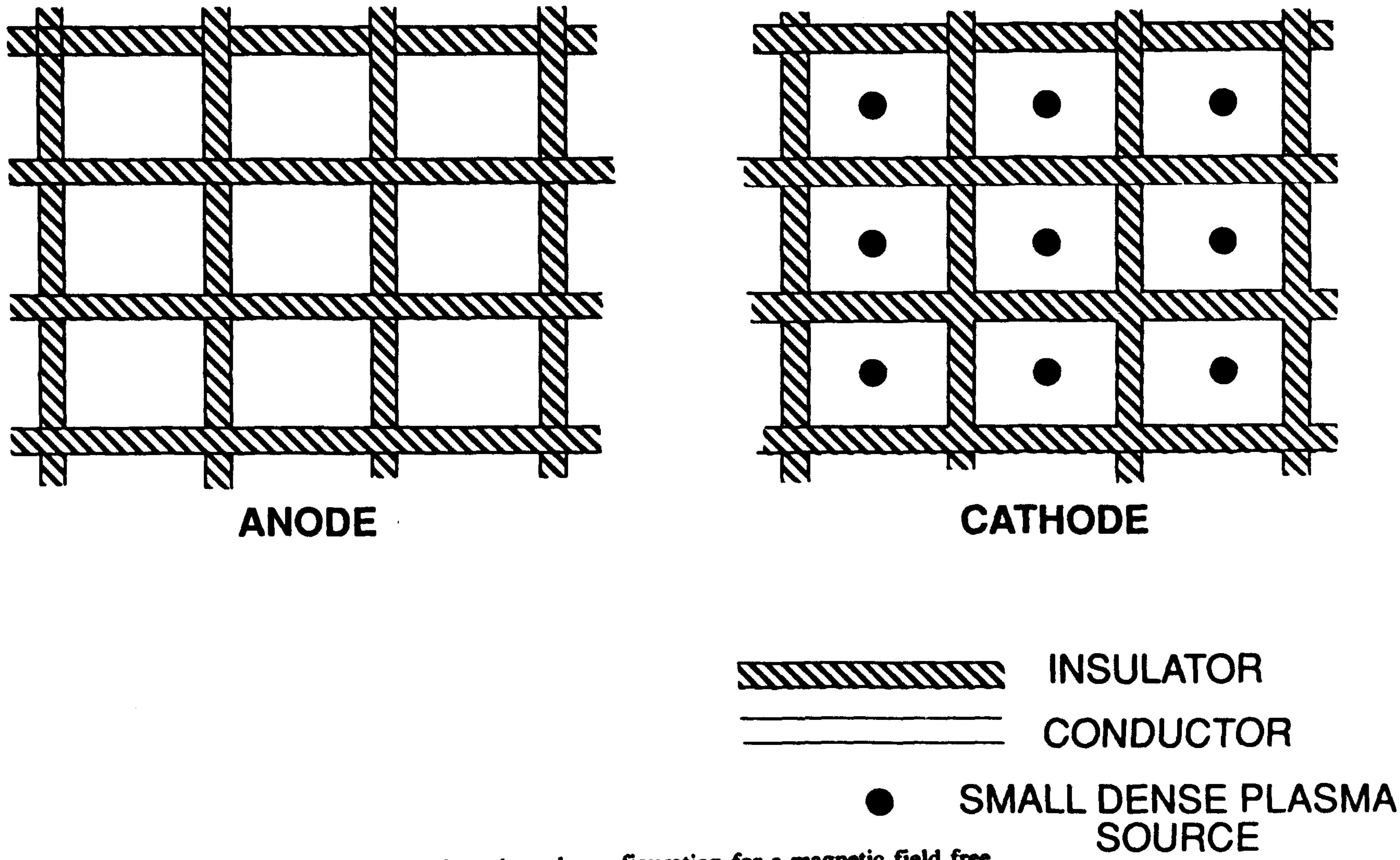

Fig. 4. A potential cathode and anode configuration for a magnetic field free agile mirror system.

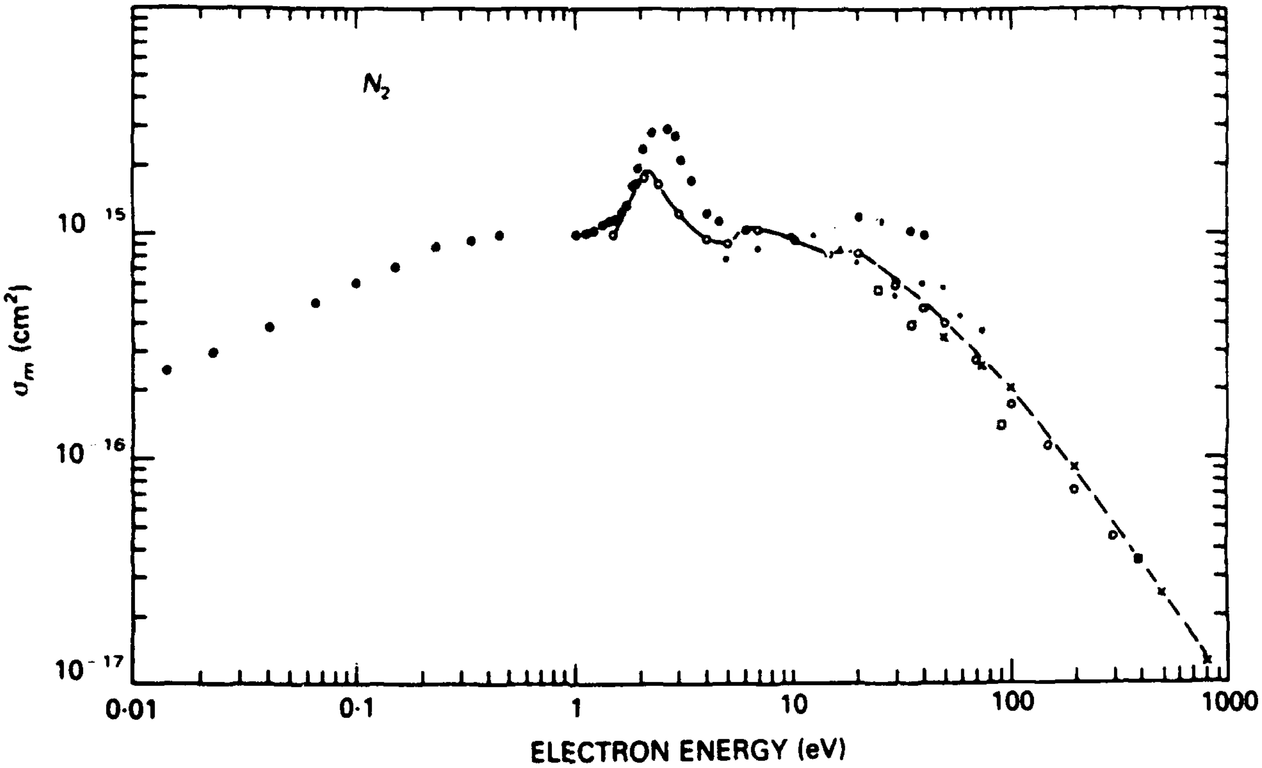

Fig. 5. The momentum exchange collision frequency as a function of electron energy for collisions with N_2.

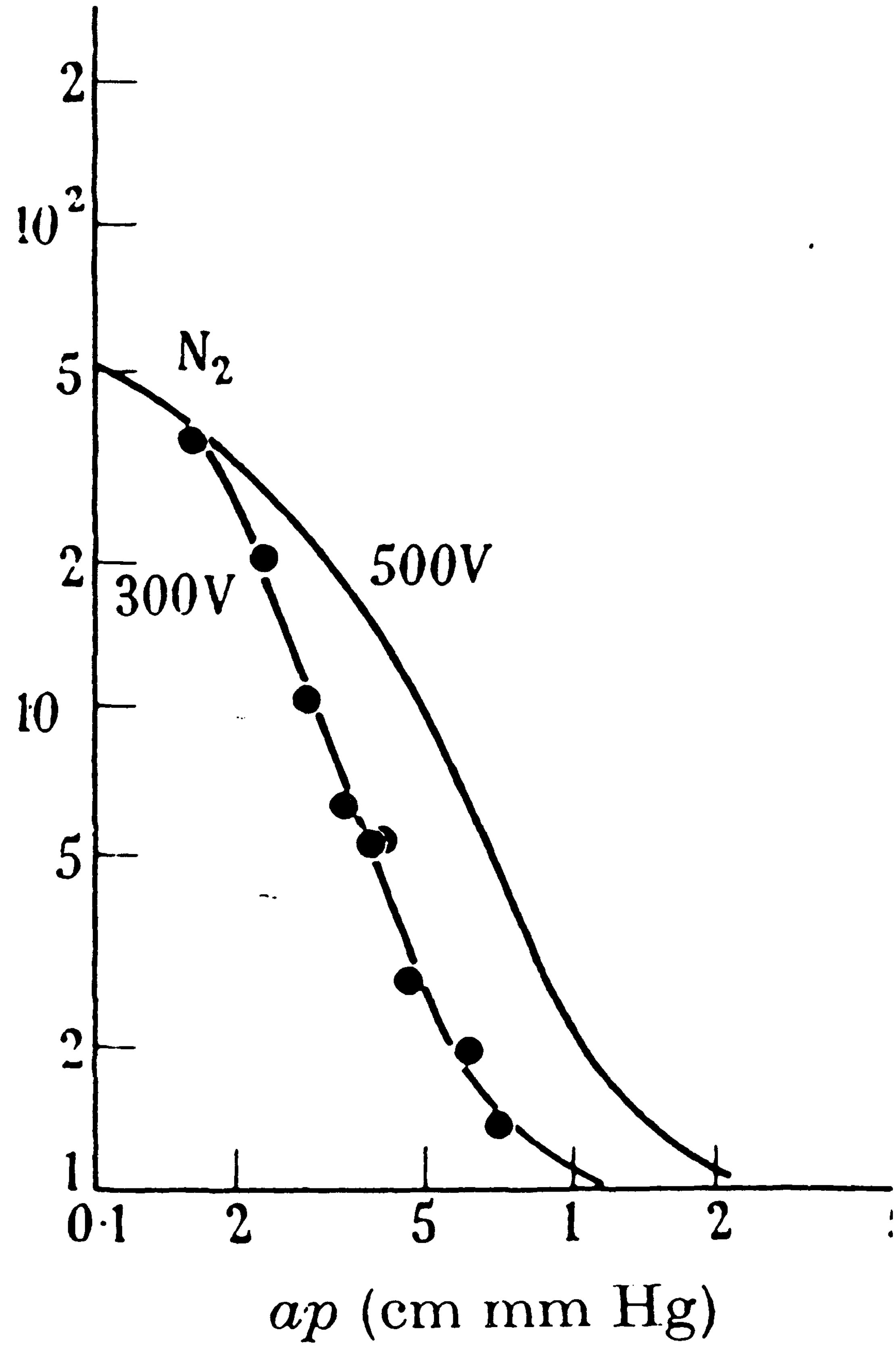

Fig. 6. The hollow cathode effect. The current as a function of pressure times separation of the two cathode plates, from Ref. (10).

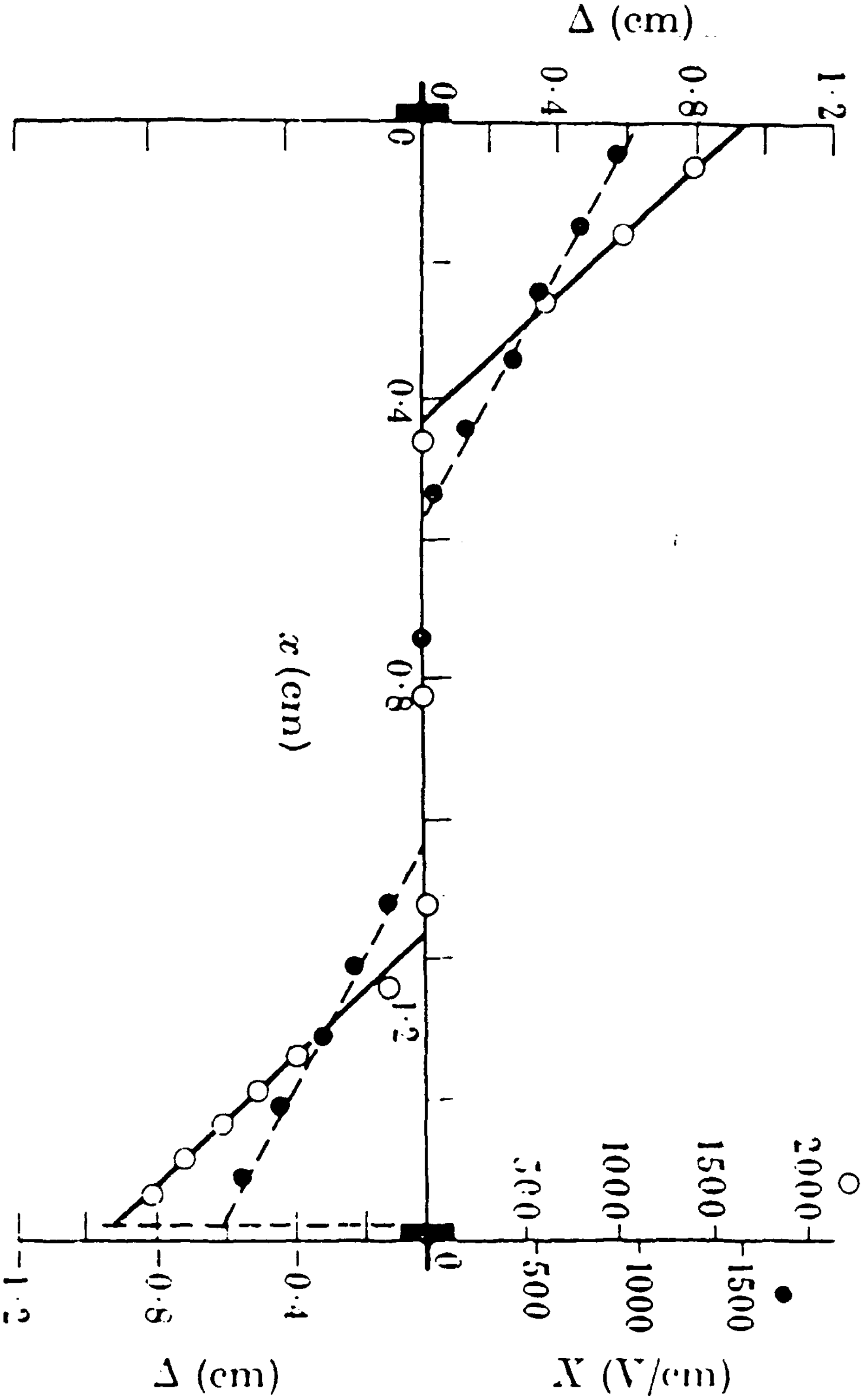

Fig. 7. The electric field (proportional to the displacement Δ of a probe beam) as a function of distance from the cathode plates, from Ref. (10).

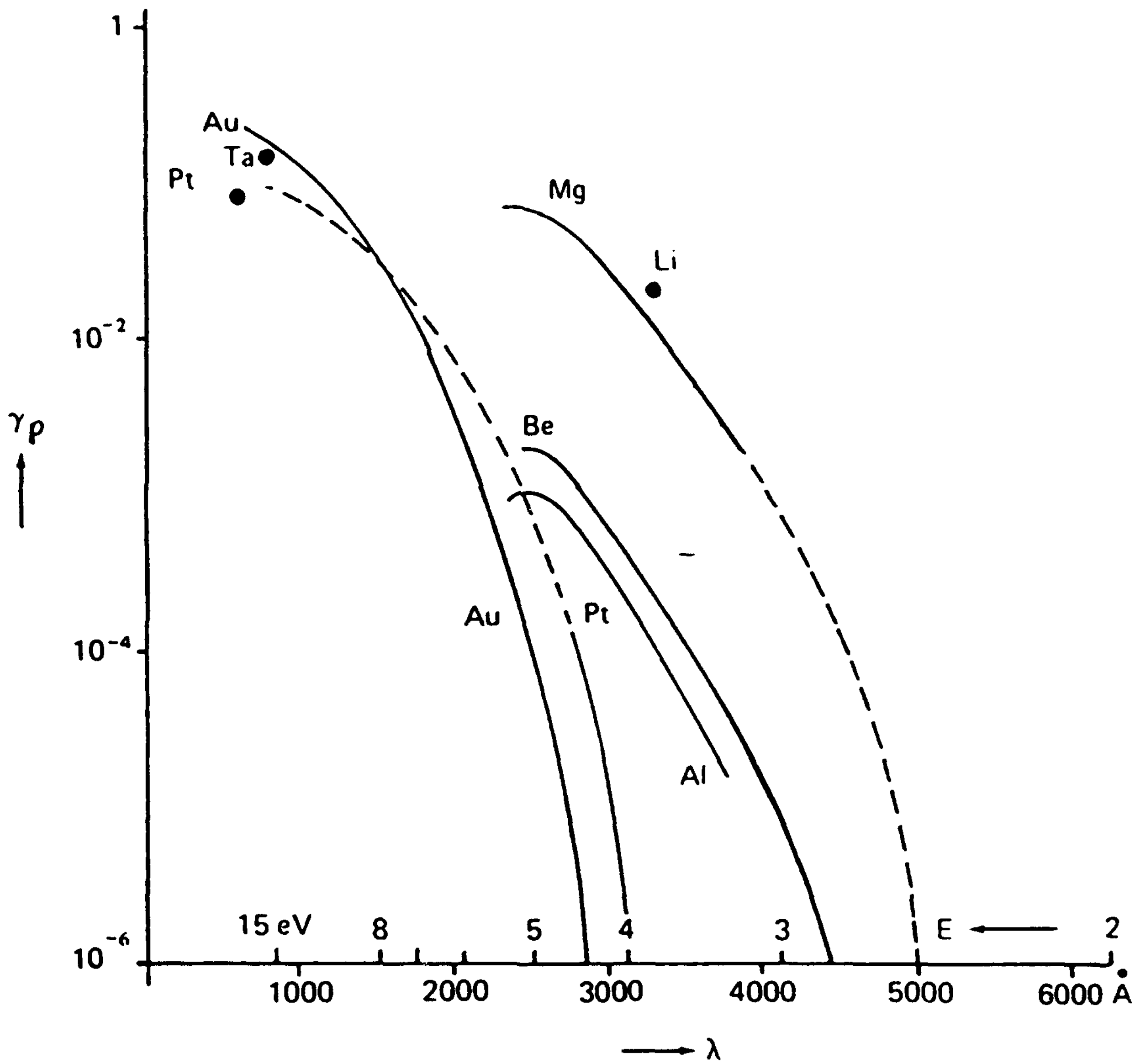

Fig. 8. The quantum efficiency for electron release as a function of photon energy, when various surfaces are emitted, from Ref. (13).

Table 1

Comparison of the properties of beam generated and thermal plasmas.

	Beam Plasma	Thermal Plasma
Density scaling	$n \alpha J^{1/2}$, $n \alpha N^{1/2}$	$n \alpha J$, $n \alpha N^2$
Voltage	Increases with length	Independent of length
Fast electrons at anode	Yes	No
Electric field in plasma	No	Yes
Metastables Required	No	Yes
Emission from	Cathode bottom wall	Cathode side wall